teen minutes. Similar studies made at the end of the exposure to environmental temperatures varying from 14° to 30°C. show a distinct tendency towards a flattening out of the curves at the higher temperatures. Thus when the thermal junction was passed down the front and back of the body over exactly the same lines and at the end of two and one-half hours' exposure, the difference between the highest and lowest points in the curves was 10.6°C. with an environmental temperature of 14.6°C.; 9.8°C. with a temperature of 19°C.; 5.4°C. with a temperature of 25.8°C.; and 4.2°C. with a temperature of 30°C.

This study of the temperature of the skin has two important bearings upon all investigations on the heat production of the human body. First, in all researches on direct calorimetry it has been necessary to corrct the heat actually measured by the calorimeter for the heat gained or lost from the body as the result of changes in temperature. Heretofore it has been assumed that as temperature curves measured either deep in the body trunk or in the artificial cavities are similar, a change of 0.1° in rectal temperature indicates a change of 0.1° in the temperature of the entire body. Our observations, particularly with cold temperature environments, show skin temperatures falling several degrees even when interior body-trunk temperatures may be simultaneously rising slightly. The correction of direct heat measurements by records of the rectal temperature is thus open to grave criticism. Unfortunately no substitute correction can as yet be offered. Secondly, these pronounced differences in skin temperature have great significance in any consideration of the so-called 'law of surface area.' It is still maintained by many physiologists that, practically independent of species, the heat production of the quiet organism is determined by its surface area. Our observations show clearly that, contrary to popular impression, the temperature of the skin, presumably one of the most important factors affecting heat loss, is very far from uniform for we have seen that even with well-clothed individuals differences in the temperature of various localities of 4° to 5°C. are of regular occurrence. These observations bring to light and establish one more serious objection to the legality of the conceptions underlying the 'body-surface law.'

ON A CERTAIN CLASS OF RATIONAL RULED SURFACES

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As is well known, ruled surfaces, or scrolls as Cayley calls them, may be generated or defined in a number of ways. There exists, for example, a one-to-one correspondence between ruled surfaces and a certain class of partial differential equations, so that the theories of the two classes are abstractly identical.

A much favored method, especially in descriptive geometry, consists in considering ruled surfaces as continuous sets of straight lines, or generatrices, which intersect three fixed curves, the directrices, simultaneously. If these are algebraic curves of orders l, m, n, with no common points, the ruled surface which they determine is, in general, of order $2\ l.\ m.\ n$.

Frequently, ruled surfaces are also defined as systems of elements, either common to two rectilinear congruences, or to three rectilinear complexes.

Of great importance for the following investigation is the definition of ruled surfaces as systems of lines which join corresponding points of an (α, β) — correspondence between the points of two algebraic curves C_m and C_n of orders m and n. If these curves are plane, and if to a point of C_m correspond α points on C_n, and to a point of C_n β points of C_m, then the order of the surface is in general $\alpha\ m + \beta\ n$.

Finally there is the cinematic method in which ruled surfaces are generated by the continuous movement of the generatrix according to some definite cinematical law. In this connection the description of the hyperboloid of revolution of one sheet is well known.

The literature seems to contain but little about this method of generating ruled surfaces. A number of treatises on differential geometry contain chapters on cinematically generated surfaces.

The class of surfaces here considered is obtained as follows: Given a directrix circle C_2 and a directrix line C_1, which passes through the center of C_2 at right angles to the plane of C_2. The generatrix g moves in such a manner that a fixed point M of g moves uniformly along C_2, while g in every position passes through C_1. The plane e through C_1 in which g lies evidently rotates about C_1 with the same velocity $k\theta$ as M. In this plane e, g rotates about M with a uniform velocity $k\mu\theta$. When $\mu = p/q$ is a rational fraction, the surface generated is also rational and belongs to the class of ruled surfaces generated by means of an (α, β) correspondence between C_1 and C_2.

When C_1 coincides with the z-axis, so that C_2 lies in the xy-plane, and we denote by ρ the distance of the projection P' of a point P on the generatrix g from the origin and by θ the angle $P'OX$, the equations of the surface expressed by the parameters ρ and θ are

$$x = \rho \cos \theta,\ y = \rho \sin \theta,\ z = (\rho - a) \cot \frac{p}{q}\, \theta.$$

It is shown that these may be expressed rationally by ρ and another parameter t. Also the implicite cartesian equation of the surface is obtained, as well as are the parametric equations of the double curve of the surface. The following theorems are of interest:

Theorem 1. The surface of the class is rational and of order 2 $(p + q)$ *or* $p + q$*, according as* q *is odd or even.*

Theorem 2. When q *is odd the generatrices of the surface cut* C_1 *and* C_2 *in two*

point sets which are in a $(q, 2\,p)$ — *correspondence.* C_1 *and* C_2 *are* $2\,p$*-fold and* q*-fold curves of the surface. The surface has moreover* p *real and* $2\,pq - 2\,p - q + 1$ *imaginary double generatrices.*

When $q = 2s$ *is even the generatrices cut* C_1 *and* C_2 *in two point sets which are in an also rational* (s, p) — *correspondence.* C_1 *and* C_2 *are respectively* p*- and* s*-fold curves of the surface. The surface has no real, but* $ps - p - s + 1$ *imaginary double generatrices.*

In the whole discussion the assumption is made, of course, that p and q are relatively prime.

Theorem 3. When q *is odd the order of each of the* $(q - 1)/2$ *double curves is* $4\,p$ or $2\,q$ *according as* $q \lesseqgtr 2\,p$. *They are rational and each lies on a surface of revolution of order 4 generated by the rotation of an equilateral hyperbola about the* z*-axis.*

Theorem 4. When $q = 2s$ *is even and* s *odd, there are* $(s - 1)/2$ *double curves of order* $2p$ *or* q *according as* $p \gtrless s$, *and one double curve of order* p *or* s, *according as* $p \gtrless s$. *When* $s = 2\sigma$ *is even, there are* σ *double curves of order* $2\,p$ *or* q, *according as* $p \gtrless s$.

The intersections of the double curves with a plane through the z-axis may be arranged according to certain cyclic groups whose orders may be easily determined. One interesting fact is that the surfaces of the class in certain species, are applicable among themselves. The following theorems appertain to this fact:

Theorem 5. Surfaces of the class are applicable to each other when their orders are $2\,(p + q)$ *and* $2\,(m\,p + n\,q)$, *and the ratio of the radii of their* C_2*'s is* m/n, *with* q *odd,* p *and* q, m *and* n, *and* m *and* q *as relative primes.*

Theorem 6. Surfaces of the class of odd order are applicable to each other when their orders are $p + q$ *and* $m\,p + n\,q$, *and the ratio of the radii of their* C_2*'s is* m/n. *Moreover* q *is even, and* p *and* m *are odd.*

As the surfaces of even and odd orders are respectively bifacial and unifacial, we have

Theorem 7. Bifacial and unifacial surfaces of the class are applicable to surfaces of the same type only.

The intersection of a torus with the surfaces of the class yields all so-called cycloharmonic curves. Also the 'bands of Moebius' may readily be cut out from the surfaces.

Among the class here considered are cubic, quartic, and quintic scrolls investigated by Cremona, Cayley, Schwarz and others. Models of these have been and are being constructed in the mathematical model shop of the University of Illinois.